Density Measurements – Background

I. Recording Significant Figures for Laboratory Data

In chemistry lab, you will measure and record *many* pieces of data. A reasonable question to ask is "how many significant figures should I record"? A simple answer is "as many as you can". Helpful, right? Well probably not, but let's look closer to see how it's actually a good rule of thumb.

If you were to place an object on a digital balance to measure its mass and the balance read:

25.046 g

you'd probably record "25.046 grams" in your notebook. This is easy, so what's the big deal? Let's consider another scenario. You placed a different object on the balance and it read:

25.000 g

What *should* you record in your notebook? Many students will record "25 grams", while others record "25.0 grams", "25.00 grams", or even "25.000 grams". Which is correct, and does it matter? A variety of explanations all produce the same answer:

- You should always record data to the *first digit of uncertainty*. What does this mean? In the case of the balance, the mass was only shown to three decimal places. The 4^{th} decimal place is completely unknown, making the 3^{rd} decimal the first digit of uncertainty. Look at it this way, the 3^{rd} decimal could truly be zero (rounded down from the 4^{th} decimal), or it could be 9 (rounded up by the 4^{th} decimal). For example, if we could see the 4^{th} decimal place, both of the masses below would round to 25.000 g on a 3-decimal balance.

  25.0004 g   24.9995 g

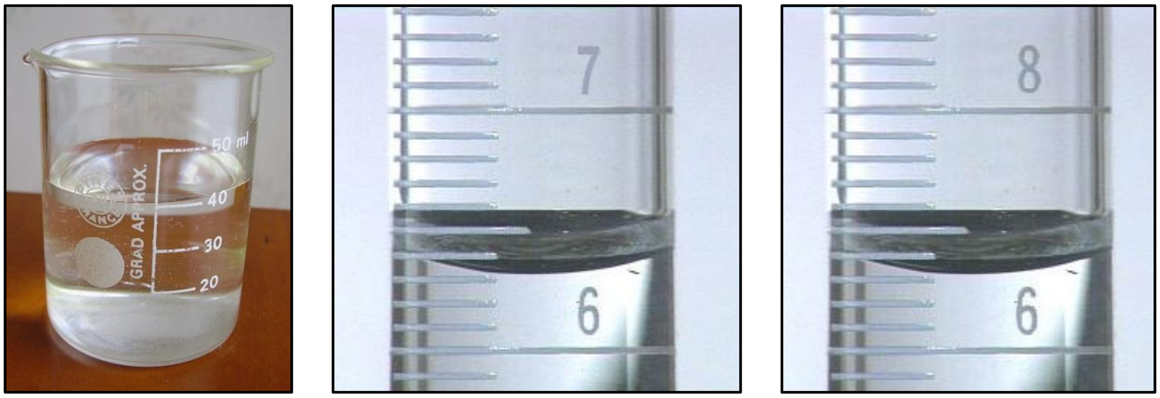
  In our 3-decimal balance example, the last digit displayed is the *first digit of uncertainty*, so once again, you should record "25.000 grams". Simply put, record every digit from a digital balance!

- If you record "25 grams", the mass could actually range anywhere from 24.500 g to 25.499 g. See the problem? An incredible amount of uncertainty is introduced when you neglect to record all digits. The balance displays the mass as 25.000 g, so that is the mass you record.

- If the balance reads 25.000 g how many significant figures are present?

  There are five sig figs in the display, so you should record the mass to five sig figs and therefore "25.000 grams". If you record the mass as "25 grams", only two sig have been recorded, and the final density can only be reported to two sig figs, at most. With only two sig figs, a false level of uncertainty has been introduced, and that is why all of this matters. Never ever drop significant figures, even if they are zeroes!

Now let's consider how many digits to record when measuring from an analog scale. For this example, we'll consider measuring volume using a beaker versus a graduated cylinder.



(a) (b) (c)

Figure 1. Volumes of water in a beaker and graduated cylinders [0.1 mL (b) and 0.2 mL (c) increments].

First, the volume of fluid is always read at the *bottom of the meniscus*, the concave curvature of the water clearly seen in the graduated cylinders. In the beaker, we can see the meniscus is slightly above 40 mL. Unfortunately, the volume graduations on the beaker are in 10 mL increments only. That means the only digit we can record with *certainty* is the 4 of 40 mL (i.e., it is certainly not 30 mL and it is certainly not 50 mL). But we can estimate a second digit, albeit with *uncertainty*. Is it 40 mL? Is it 41 mL? Is it 42 mL? That is up to your judgement, and therefore the second digit is the *first digit of uncertainty*. In this case, it is unreasonable and incorrect to record more than two significant figures (e.g. "41.3 mL") as the beaker's graduations do not allow for that level of certainty. Perhaps this example more clearly demonstrates the concept of the "first digit of uncertainty".

How many digits can be recorded for the volume shown in each graduated cylinder? Notice the graduations in Figure 1b are 0.1 mL increments, while those in Figure 1c are 0.2 mL increments. In Figure 1b, the bottom of the meniscus lies between 6.3 mL and 6.4 mL, but we should estimate one digit further to 6.3_ mL with the second decimal as the first digit of uncertainty. The volume of Figure 1b should be recorded to three significant figures, so we'll say 6.31 mL.

The volume of Figure 1c is a little trickier. The meniscus lies between 6.6 mL and 6.8 mL, but there is no graduation for 6.7 mL. If, and only if you can mentally create a "split increment" at (in this case) 6.7 mL, and see that the meniscus lies beneath it, can you report the volume as 6.6_ mL, estimating the last digit. The ability to create the split increment is contingent on the visual degree of spacing between the given increments. In Figure 1c, the meniscus is crystal clear and the picture is extremely zoomed, allowing us to confidently record the volume as 6.62 mL.

The clarity of Figure 1c is likely not realistic however. What happens if you can't, with certainty, discern where the meniscus lies between 6.6 mL and 6.8 mL? If you are uncertain about the first decimal, then that is your first digit of uncertainty and you'd stop there. Report the volume to only one decimal place and two sig figs (6.6 mL, 6.7 mL, or 6.8 mL).

II. Percent Error and Standard Deviation

Another very important, and related, aspect of this experiment is to learn to apply the measures of *accuracy* and *precision* to **all** quantitative laboratory measurements. Accuracy is a measure of the agreement of a measurement with its **accepted** value. Usually, it is expressed as a percent error, which is defined by the equation:

$$\%\,\text{error} = \frac{\text{measured value - accepted value}}{\text{accepted value}} \times 100$$

Accepted values for many properties are found in the *Handbook of Chemistry and Physics* or published in some other reputable source, such as textbooks or online. Any accepted value used in your report must be accompanied by a proper citation to the reference. The precision of a measurement is a measure of the mutual agreement of repeated determinations; it is a measure of the reproducibility of an experimental value. The most useful measure of precision is the standard deviation (*std*). This is the range of values within which 68% of all measured values are likely to fall. The smaller this range, the more precise are the results. Standard deviation is given by the following formula:

$$std = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}}$$

where N is the number of measurements and $\bar{x}$ is their average. An example is shown below:

- A student makes four measurements of the density of an object and obtains the values 10.10 g/cm^3, 10.20 g/cm^3, 10.12 g/cm^3, and 10.15 g/cm^3.

i	x_i (g/cm^3)	$\lvert x_i - \bar{x} \rvert$ (g/cm^3)	$(x_i - \bar{x})^2$ (g/cm^3)^2
1	10.10	0.04	16×10^{-4}
2	10.20	0.06	36×10^{-4}
3	10.12	0.02	4×10^{-4}
4	10.15	0.01	1×10^{-4}
	$\bar{x} = 10.14$		sum = 57×10^{-4}

$$std = \sqrt{\frac{57 \times 10^{-4}}{4-1}} = 0.04\ \text{g/cm}^3$$

These results would be reported as 10.14 ± 0.04 g/cm^3. The smaller the standard deviation, the more closely each individual measurement agrees with the average value and the better the precision.

Remember, all entries in your lab book must be made, in ink, directly in your tables, in your book, before you leave the lab. All measurements made in the laboratory must be reported with an appropriate number of significant figures and with the correct units. This is a very important part of this experiment and all of the experiments which follow. It will always be a factor in the grade you receive for your lab report! It is not unusual to make measurements that give very suspicious results. A suspicious measurement can be ignored, if there are other good measurements and if there is reason to believe it is bogus. However, you must not remove or delete it from your report; simply draw a line through it and indicate why you are not using it for your average value. Just because it is not the result you think you should get is not a sufficient reason. You might consider putting a copy of this paragraph at the top or each experiment in your notebook. It is that important.

III. Graphical Determination of Density

The density of any substance can be determined graphically using a linear fit of the data according to the following derivation. The equation for density is:

$$\text{density} = \frac{\text{mass}}{\text{volume}} \tag{1}$$

Rearranging equation (1) to solve for mass gives:

$$\text{mass} = (\text{density})(\text{volume}) \tag{2}$$

which can be compared to the equation of a straight line

$$y = (m)(x) + b \tag{3}$$

where *m* is the slope and *b* is the y-intercept. Comparison of equations (2) and (3) shows that if we plot the mass of a substance (y-axis) versus its volume (x-axis), we should obtain a straight line which passes through the origin (b = 0), and which has a slope equal to the density of the substance (slope = Δmass / Δvolume). In the experiment you will determine the mass of various objects, and then graph the mass versus the volume. The slope of the best fit line (determined by linear regression) through the points will be equal to the density. Spreadsheet programs, like *Microsoft Excel*, can be used to make a scatter plot with your data and then fit the data with a linear trendline using a regression routine. The advantage in using this graphical technique is that it weights individual data points based on their relative fit to a theoretical trendline, whereas a simple average equally weights all data points.

Density Measurements – Procedure

Purpose: There are two parts to this experiment, each with their own purpose.

Part 1: Determine the density of water with a focus on measurement uncertainty and its effect on significant figures.

Part 2: Graphically determine the density of a metal.

Procedures:

Part 1. ***Determining the Density of Water***

Obtain a small beaker (30, 50 , or 100 mL) from your drawer and record its mass. Record every digit from the balance, even trailing zeroes as they are significant figures (see Background). Pour a volume of water into the beaker and record its volume. How many significant figures should you record for the volume? (See Background.) Record the temperature of the water in your beaker, as density varies with temperature. How many significant figures should you record for the temperature? (See Background). Lastly, record the combined mass of the beaker and its water.

Discard the water, dry the beaker, and repeat the procedure twice more for a total of three trials. You do not need to use the same volume of water for each trial. In fact, it is a good idea to use different volumes to demonstrate how density is not dependent on a *single* variable. As part of your final results, you will be asked to calculate a percent error for your average density of water. To calculate a percent error, you will need to record the accepted density of water at the experimental temperature. Consult a reputable literature source (CRC Handbook of Chemistry and Physics) for the temperature-dependant accepted density of water.

Repeat the entire procedure above, but this time use a 10 mL graduated cylinder to hold the water and measure its volume. Make sure to perform three trials. Between trials, dry the inside of the graduated cylinder as much as possible; a neatly rolled paper towel should suffice. How many significant figures should you record for the volume using a 10 mL graduated cylinder? (See Background.) Will more significant figures lead to higher precision and accuracy of the experimental average density of water?

Part 2. ***Graphical Determination of the Density of a Metal***

Obtain a set of five metal rods and record the metal ID in your notebook. As you perform this experiment, keep the rods in your set together; do not mix-and-match rods with other sets! Obtain the mass of one of the five metal rods, and record the value in your notebook with the correct number of significant figures. Determine the volume of that metal sample by displacement of water, as follows:

- Fill a 100 mL plastic graduated cylinder with about 50 mL of water.
- Record the exact initial volume of water in the cylinder.
- Immerse your object into the water, *being careful not to allow it to drop forcefully against the bottom of the cylinder*. Gently tap the cylinder to dislodge any air bubbles from the object and/or water droplets from the inside wall of the cylinder.
- Record the final volume of water.

The difference in the water levels before and after the immersion of the sample is the volume of the water displaced by the sample, i.e., the volume of the unknown sample. Record the temperature of the water. When done, dry the metal rod and return it to its bag. Repeat the procedure with the remaining four metal rods, recording all of the data as you go. Is it necessary to dry the graduated cylinder between each trial? Record the *accepted* value for the density of your metal.

Calculations: In this section of your notebook, you must show all calculations, in addition to any Excel calculations, using the correct number of significant figures and including all units. For each Part below, follow the steps to make sure you have done everything required.

Part 1. ***Determining the Density of Water***

1) Calculate the mass of water in the beaker.
2) From the calculated mass and the recorded volume, calculate the density of water.
3) Repeat for trials 2 and 3.
4) Calculate the average density of water.
5) Calculate the standard deviation of your average density.
6) Using your accepted literature value for the density of water, calculate the *percent error* in your experimentally determined average density.

$$\boxed{\%\ \text{error} = \frac{\text{measured value - accepted value}}{\text{accepted value}} \times 100}$$

7) Repeat all calculations for the density trials using the 10 mL graduated cylinder.

Part 2: ***Graphical Determination of the Density of a Metal***

1) Calculate the volume of water displaced by each metal rod.
2) In Excel, make a plot of volume (x-axis) versus mass (y-axis) with a linear trendline, its equation, and the R^2 value (see Background). The R^2 value is a measure of how closely your data fits the straight line. R^2 can range from 0-1, with values closer to 1 signifying a good linear fit for the data, and where values <1 suggest a poor linear fit. In other words, it is a measure of the trustworthiness of the linear fit. An R^2 value <0.9 may signify the data is not linear, too scattered, or generally not trustworthy and becoming more pronounced as R^2 approaches zero.
3) Using your Excel plot, determine the density of the metal.
4) Using your accepted literature value for the density of your metal, calculate the *percent error* in your graphically determined density.

Conclusions: Summarize and interpret your **key** results in well-organized paragraphs. Be sure to address the following points.

- Compare the experimentally determined density of water to that taken from the literature. Discuss the precision of densities from the individual trials, as shown by the standard deviation, and the accuracy of the average density, as shown by the percent error. Did the use of a 10 mL graduated cylinder lead to higher accuracy? What are some possible sources of error?
- Discuss the density obtained for the metal rods. Report the percent error in your result and offer some reasons as to why your experimental value differs from the accepted value.